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The Dynamic Intervention Game

Intergenerational Transfer of Risk of Nuclear Proliferation

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EXECUTIVE SUMMARY:

This note contains a description and solution of a simple problem modeling determination of the optimal time for intervention, by the international community, in the case of a rogue state believed to be engaging in nuclear proliferation. Illustrative examples are provided, and both challenges and opportunities in obtaining the putative problem data are discussed.

The purpose of this note is to present the solution of the following problem, termed here as "the dynamic intervention game," along with examples of various cases of the solution. The international community is contemplating intervention to prevent a rogue state from engaging in proliferation of nuclear weapons technology. The community assesses that the probability per unit time of the state proliferating is $\lambda > 0$. The consequences of a successful proliferation are C_P, measured on some scale, at the time at which the proliferation occurs. The consequences of an intervention, again at the time the intervention occurs, are C_I, measured on the same scale as the consequences of the proliferation. If all consequences are discounted to their present value at rate $\gamma \geq 0$, then at what time should the international community intervene in order to minimize the expected net present value of the associated consequences?

The solution of this problem is as follows: The expected net consequence of an intervention at time *t* is

$$C(t) = C_P \int_0^t \lambda e^{-(\lambda+\gamma)\tau} d\tau + C_I e^{-(\lambda+\gamma)t} = C_P \left\{ \frac{\lambda}{\lambda+\gamma} \right\} \left(1 - e^{-(\lambda+\gamma)t}\right) + C_I e^{-(\lambda+\gamma)t}. \quad (1)$$

Explanation: The probability that proliferation occurs between τ and $\tau + d\tau, 0 \leq \tau < t,$ is $e^{-\lambda\tau}(\lambda d\tau)$, where the first factor is the probability proliferation does not occur prior to time τ, and the second is the probability that proliferation occurs in $d\tau$ at τ, given that it does not occur prior to that. The appropriately discounted contribution to the net present value of the consequences from any such proliferation, given that it occurs, is $C_P e^{-\gamma\tau}$. The expected contribution to the net present value of the consequences from such events is then $e^{-\lambda\tau}(\lambda d\tau) C_P e^{-\gamma\tau}$. The sum over all such contributions then is, in the limit as $d\tau \to 0$, the first term on the right-hand side of (1). The probability that proliferation does not occur prior to time t is, as already observed, $e^{-\lambda t}$. If this happens, then the corresponding contribution to the net present value of the consequences is $C_I e^{-\gamma\tau}$. The second term on the right-hand side of (1) is the product of these last two factors, and hence the expected contribution to the net consequences from interventions (assumed to occur at time t).

Therefore

$$\left.\frac{dC}{dt}\right|_t = C'(t) = [C_P \lambda - C_I(\lambda+\gamma)] e^{-(\lambda+\gamma)t},$$

which has the sign of $C_P\lambda - C_I(\lambda+\gamma)$. It follows that if $C_P\lambda < C_I(\lambda+\gamma)$, then the expected net present value of the consequences of (would-be) intervention at time *t* is a strictly decreasing function of *t*. Therefore, in this case, that expected net value is always reduced by delaying intervention. The optimal time of intervention is therefore never, and the optimal value of the associated net present value of the consequences is

$$C(\infty) = \lim_{t\to\infty} C(t) = \frac{C_P \lambda}{\lambda + \gamma}. \qquad (2)$$

On the other hand, if $C_P \lambda > C_I(\lambda + \gamma)$, then the expected net present value of the consequences increases as a function of the prospective time of intervention. In this case the optimal time of intervention is therefore $t = 0$, and the optimal value of *C* is $C(0) = C_I$.

Example: Suppose $C_I = 1$, in some units, and $\gamma = .02, \lambda = 0.1$, both per annum. Values of the consequences of intervention, in the same units as the consequences of proliferation, will be selected corresponding to each of the two cases distinguished above, which can be thought of as respectively

$$\frac{C_P}{C_I} < \frac{\lambda + \gamma}{\lambda} \text{ and } \frac{C_P}{C_I} > \frac{\lambda + \gamma}{\lambda}. \qquad (3)$$

The “boundary” value is $(\lambda + \gamma)/\lambda = 0.12/0.1 = 1.2$. We therefore consider the two values $C_P = 0.5 \text{ and } 2$, which correspond respectively to the two ranges of C_P / C_I delineated in (3). These can be thought of, again respectively, as the ranges of slight expected consequences of proliferation and of significant expected consequences of intervention.

Figure 1 is a plot of the expected net consequence curves for these two cases, over a period of 50 years. By this time proliferation is almost certain to have occurred, so the net expected consequences should be very near the asymptotic value (2). In these two cases the costs of proliferation and of intervention are closely comparable (factor of four), as therefore are the two optimal net expected consequences (factor of two). Rhetoric associated to decision making regarding possible intervention often suggests evaluations of the two types of consequences that differ by significantly more than that. Figure 2 illustrates that, for the above parametric values, but now $C_P = 0.1 \text{ and } 10.$

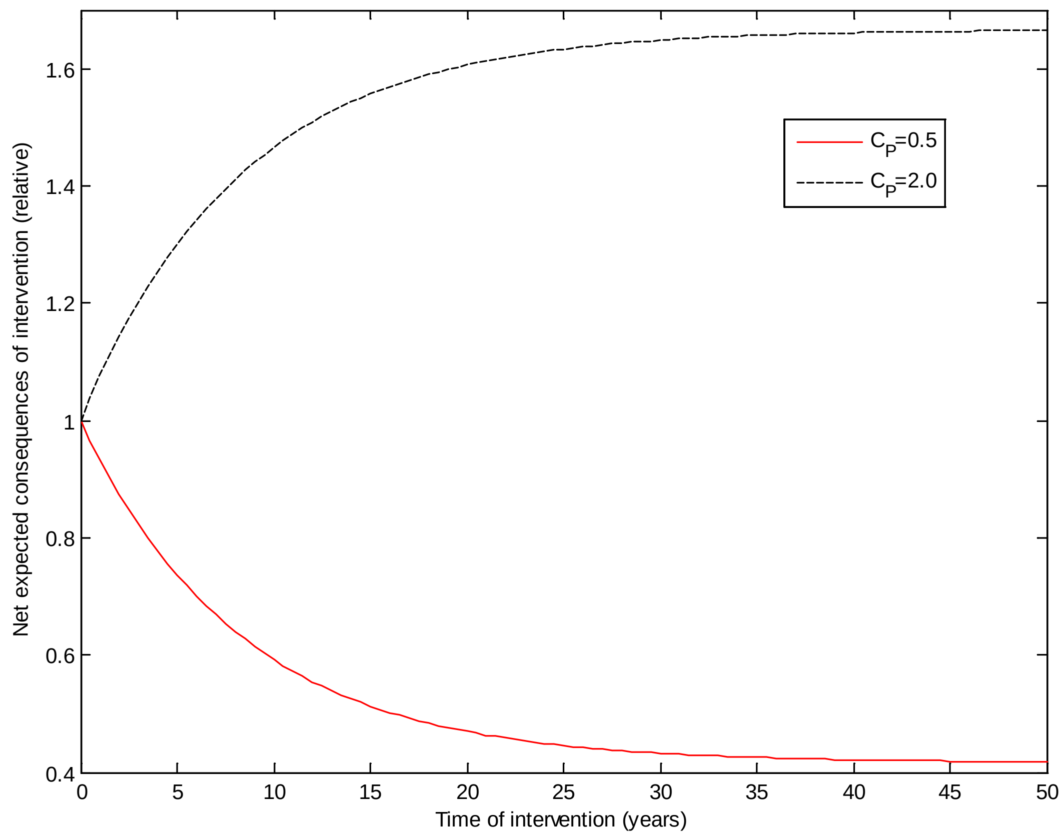

Figure 1 - Expected net consequences, as a function of time of intervention, for $C_I = 1, \gamma = .02, \lambda = 0.1,$ and the indicated values of C_P.

The optimal time of intervention is determined by (3), which depends on only two parameters: C_P / C_I, the value of the instantaneous consequences of proliferation, relative to those of intervention; and γ / λ, the discount rate relative to the rate of proliferation. (The right-hand side of the inequalities in (3) can be written as $1 + \gamma / \lambda$.) Large values of the first of these parameters mean proliferation is deemed to have greater consequences than intervention, and thus militate toward earlier intervention. Larger values of the second mean consequences to later generations are being discounted more rapidly, relative to the rate at which proliferation is deemed likely to occur, or that proliferation is occurring less rapidly, relative to the rate of proliferation. From either perspective, such a change intuitively will decrease the urgency of immediate intervention.

Any evaluation of the essential parameters, C_P / C_I and γ / λ, likely will be based on evaluations of the individual parameters, γ, λ, C_P and C_I, in the order that we find it convenient to discuss them. Values for these

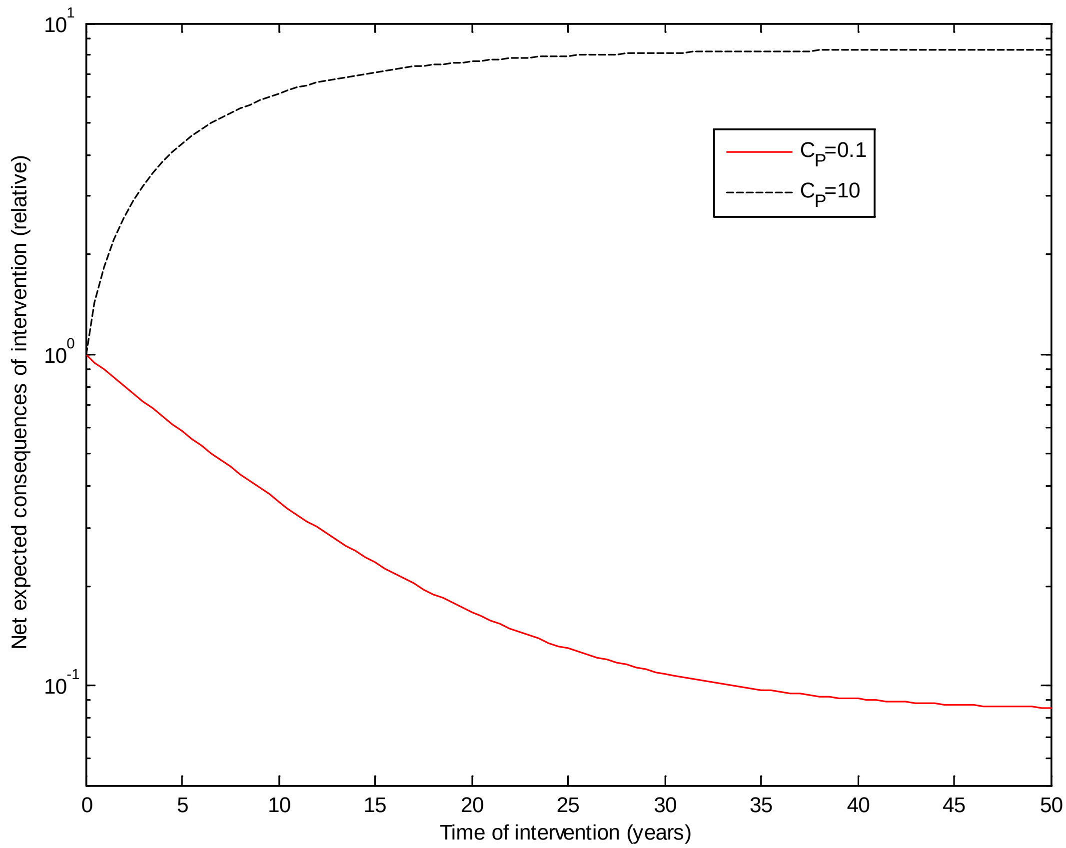

Figure 2 - Expected net consequences, as a function of time of intervention, for $C_I = 1, \gamma = .02, \lambda = 0.1,$ and the indicated values of C_P. (Note the logarithmic vertical scale.)

parameters derive from vastly different fields, but they all are subject to significant uncertainty. For example γ represents the rate at which it is deemed acceptable to transfer risk to future generations. The subject of intergenerational transfer of risk, particularly whether *any* degree of it is acceptable, has been extensively discussed, especially from the viewpoint of ethics. See, for example, (Okrent & Pidgeon, 2000), and the various contributions in the special issue of *Risk Analysis* introduced by this paper.

The proliferation rate can be considered as determined by two considerations: the time until a state decides to proliferate, and the time that will be required to execute that decision, once reached. The former is "intent," and its determination - or at least the determination of motivation to decide on intent - can be viewed as one possible use of the methodology sometimes termed as Quantitative Empirical Analysis (Fuhrmann & Kreps, 2010), (Subbaiah & Nelson, 2015). The latter is "capability," and a variety of proposed methodologies for assessing technological capabilities seem to have bearing on its estimation; cf., e.g., (Kwon & Il, 2009), (Singh & Way, 2004), (Sweeney & Charlton, 2013). Singh & Way in particular specifically seek to estimate statistically the instantaneous proliferation hazard rates of states, based upon a variety of national characteristics. To be sure the details of the methodology employed have been criticized (Montgomery & Sagan, 2009); nonetheless it is clear that similar methodologies can be employed to estimate statewise proliferation rates, precisely as required by the dynamic intervention game.

What then can be said about the evaluation of the two types of consequences, C_P and C_I, and the associated uncertainties? At a minimum the consequences of any additional proliferation would be further damage to the nonproliferation regime, particularly the NonProliferation Treaty; some see this as catastrophic, perhaps especially because to them the nonproliferation regime already seems fragile (Allison, 2010). Aside from damage to the nonproliferation regime, at a minimum any additional proliferation seemingly would threaten any stability existing in the region where it occurred, and certainly there are cases of very real concerns of threats of immediate nuclear attack. Certainly there is huge room for differing estimates of the consequences of any particular instance of suspected proliferation, and in fact likely huge differences in those consequences for different individuals and different states. This certainly will be a challenge to obtaining any international consensus for intervention, especially for any intervention going beyond "mere" economic sanctions.

Fortunately most cases of interventions that have occurred thus far have been based on such sanctions. This is certainly significantly less costly than the alternative of military intervention, albeit at the cost of a level of success perhaps best described as indifferent. There have been a few cases of limited, and therefore inexpensive, military intervention. These were limited because they were directed at essentially point targets, which was feasible because they occurred early in the development of the suspected effort to build a nuclear weapons program, or at least latent capability for such a program. Their apparent success arguably confirms the optimality of early intervention that is suggested above. On the other hand, the single largest instance of military intervention in a case of suspected proliferation not only is today considered to have been based on erroneous suspicion, but also reasonably can be considered to have now, over a decade later, an as yet untold cost.

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