Optically Thick Heterogeneous Object Imaging Using Constrained Optimization and Diffusion Theory

NUCLEAR ENGINEERING

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In the field of nuclear and global security, smuggling of special nuclear materials by transportation in containers on boats poses strong threat of special interest. To prevent this possible smuggling pathway, a detection system must be implemented that will have the ability to detect high enriched uranium (HEU) where current detection systems cannot. Due to self shielding and long half lives, uranium can be hard to detect through conventional methods. A possible method of detection would be an active neutron imaging technique which would involve incident beams of neutrons upon the cargo container and neutron detectors surrounding the container. Using these detector readings and a constrained optimization technique, reconstructions of the material parameters inside a container can be performed to determine the contents. This is done by minimizing a cost function which is the difference between the boundary detector measurements and the boundary neutron fluxes computed from the inferred material properties inside the cargo. While many sets of material parameters have the ability to reconstruct the outer detector readings, constraints upon these must be applied. The valid constraint here will involve conservation and interaction physics of the neutrons in the container, thereby limiting the solution of material parameters to a realistic case.

Optimization:

When an iterative solution is considered, a parameter called the misfit is introduced which represents the iterative solution's distance from the true solution over the portion of $\partial\Omega$ where measurements are made. The objective of the optimization problem is to minimize this misfit. In the problem at hand, the misfit is the difference between the iterative solution at the boundary and the detector readings.

$$\text{misfit} = \frac{1}{2}\int_{\substack{\text{portion of }\partial\Omega\text{where}\\ \text{measurements are made}}} [\Phi - z]^2$$

This can be classified as an optimization problem because the difference between the computed iterative solution at the boundary and the neutron detector readings must be minimized while satisfying the neutron transport or diffusion equation. The equations derived from the optimization process are nonlinear, naturally requiring a descent method to solve them. This problem is very ill-posed because the neutron fluxes and the material parameters upon which the neutron fluxes depend on through nonlinear equations are both unknown. While this nonlinear problem is very ill posed, application of iterative methods cannot guarantee convergence for any realistic initial guess.

$$\begin{aligned} -\nabla\cdot D(\vec{r})\nabla\Phi + \Sigma_a(\vec{r})\Phi &= Q(\vec{r}) \ \text{ in } V \\ \frac{\Phi}{4} + \frac{D(\vec{r})}{2}\partial_n\Phi &= J^{inc}(\vec{r}) \ \text{ in } \partial V \end{aligned}$$

In the finite element setting this becomes:

$$\left[A_D + A_\Sigma + \frac{1}{2}M_{\partial V}\right]\Phi = A\Phi = F$$

with A_D the stiffness matrix, A_Σ is the mass matrix, $\frac{1}{2} M_{\partial V}$ is the boundary mass matrix, F is the *RHS* contained the contributions for the volumetric source Q and the incoming current J^{inc}. If the entire boundary is used for measurements, then $M_{\partial V} = M_{meas}$. If only Σ is to be determined, then the constrained optimization problem seeks to minimizing the following Lagrangian functional for multiple experiments:

$$\mathcal{L}(\Phi,\lambda,\Sigma) = \sum_{i=1}^{I}\frac{1}{2}[\Phi_i - z_i]^T M_{meas}[\Phi_i - z_i] + \lambda_i^T[A\Phi_i - F_i]$$

(the governing equation acts as a constraint and the adjoint flux λ is the Lagrange multiplier). From the theory of constrained optimization, we know that the optimum satisfies the following optimality conditions for $\mathcal{L}$:

$$\partial_{\Phi_i}\mathcal{L} = M_{meas}[\Phi_i - z_i] + A^T\lambda_i = 0 \quad \forall i\in I$$

$$\partial_{\lambda_i}\mathcal{L} = A\Phi_i - F_i = 0 \quad \forall i \in I$$

$$\partial_\Sigma\mathcal{L} = \sum_{i=1}^{I}\lambda_i^T(\partial_\Sigma A)\Phi_i = 0.$$

Importance of Measurement Location:

In order to provide an example demonstrating the value of multiple experiments, we consider a two-region domain, say, left and right, as shown in Figure 1. Since there are only two unknown parameter, Σ_{left} and Σ_{right}, the misfit plot can easily be represented as a surface plot as a function of Σ_{left} and Σ_{right}. *The true* solution is computed and serves as a synthetic measurement *z (in future test, to increase the* representativity, a mix of background noise and measurement noise could be added to obtain a synthetic measured values.) Then, Σ_{left} *and* Σ_{right} *are* incrementally modified to generated estimated values Φ *and* the misfit is graphed as a function of Σ_{left} *and* Σ_{right}. Three cases were run: (i) a neutron source is incident on the left face only. Figure 2(a), (ii) a neutron source is incident on the right face only, Figure 2(b), and (iii) a neutron source is incident on both the left and right faces, Figure 3. This was done to establish the importance of having incoming fluxes from multiple directions.

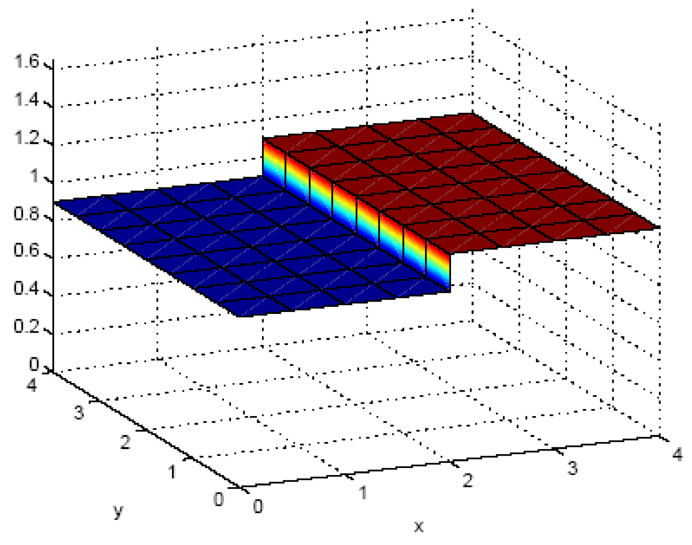
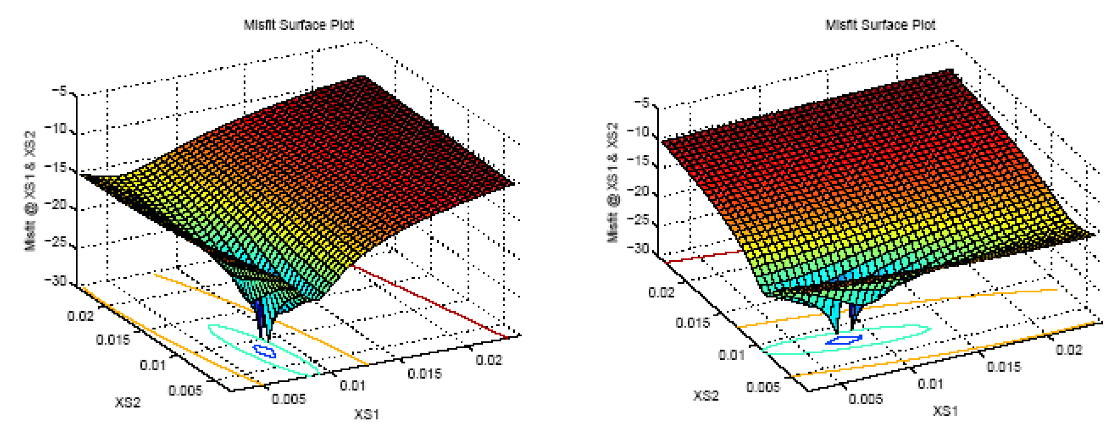


Figure 1. Example of a two region domain

In Figure 2(a), an elongated valley is produced in the direction of the cross section whose side was not illuminated. This means that the cross section in this part of the domain can vary greatly with respect to the other without significantly changing the misfit value and hence cannot be estimated with accuracy, whereas the cross section of the portion illuminated by the beam can be accurately reconstructed. If the only incident neutron beam was moved to the side of the domain, an elongated valley is produced in the direction of the opposite cross section as shown in Figure 2(b).

Again, the cross section in the region that was not illuminated can vary greatly without significantly changing the misfit, leading to a poor reconstruction.



(a) Left Incidence Only (b) Right Incidence Only

Figure 2. Misfit Surface Plots for Variable Incident Neutron beams

When both beams are on, one shining on each region side, a much smoother cone shape surface is produced as shown in Figure 3. In such a case, it will be significantly easier to determine the cross section for both regions. In the next examples, a fully position-dependent cross section is employed (i.e., the model is general enough to allow arbitrary Cartesian maps of parameters.) The insight acquired here in this simple 2-parameter model, where the misfit surface changes from elongated ellipses to rounder valley with the addition of incoming beams, applies also to the more complex model we present next, although the misfit behavior with the more complex model cannot be shown visually. In the general case, we are seeking a misfit minimum in multi-dimensional space and increasing the number of incoming beams (and thus measurements) is expected to play a critical role in our ability to find such a solution.

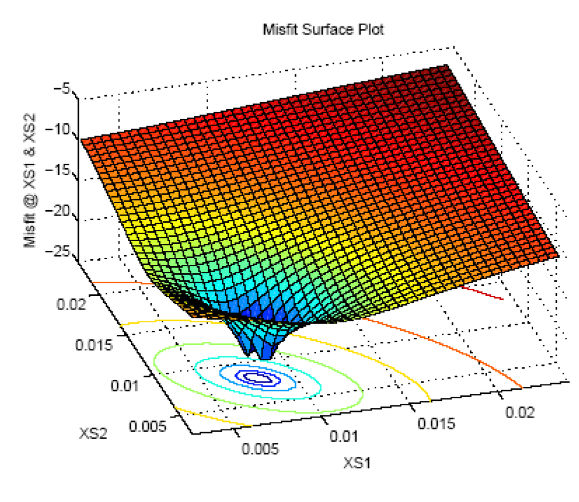
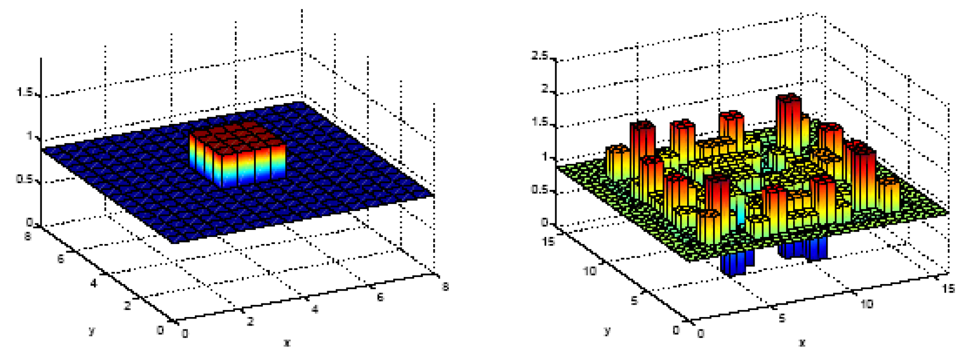


Figure 3. Both Directions

Results:

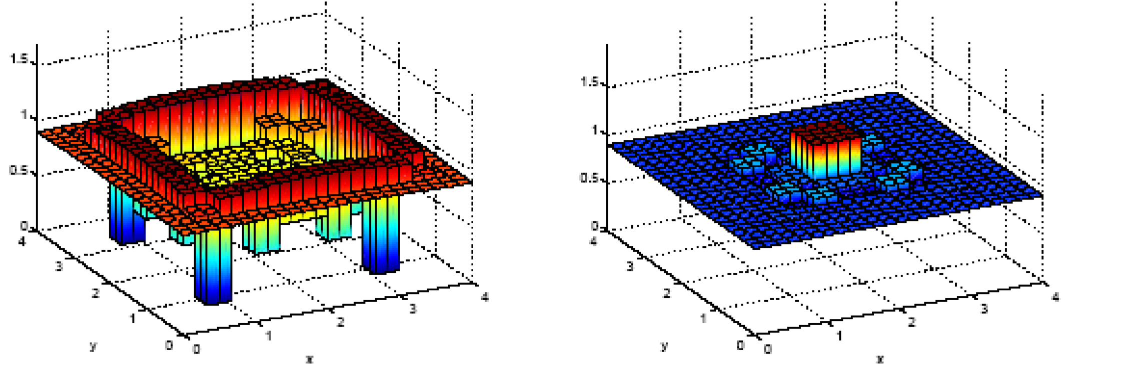
A test consisting of a strong absorber centered inside a given domain is considered. Two different domain sizes are employed to test the ability to reconstruct deep inside a domain using only boundary information. One domain was of size 8 cm $\times$ 8 cm (the cross section were such that the domain size, in diffusion length, was also 8 $\times$ 8) and the another domain was 16 cm × 16 cm with (and a size, in diffusion lengths, of 16 $\times$ 16.) This problem consisted of 8 simultaneous experiments that consisted of incoming neutron beams of equal width over an eighth of the domain each. Fig. 4 presents the reconstructed cross sections for this example. The optically thicker a domain is, the larger the error in the reconstructed cross sections. In the thicker domain case, very little information about the center of the domain (located at about 7-8 diffusion lengths from the boundaries) reaches the boundary. Such a large attenuation of information leads to a poor reconstruction.



(a) 8 cm × 8 cm (b) 16 cm × 16 cm

Figure 4. Reconstructed cross sections, strong centered absorber case

To test the resolution limits, the previous problem was modified by reducing the domain size to 4 cm $\times$ 4 cm size and placing a much smaller absorber in the center of the domain. The mesh size for the cross section reconstruction was also reduced. This reconstruction was performed twice, once with one experiment and once with 8 experiments. Fig. 6 presents the reconstructed cross sections for this example. While the system was still able to significantly reduce the optimality conditions as in the previous cases, in the case with one experiment, the regions were simply too optically thin. The cross sections in these small material zones can change by a significant quantity, from too high to too low, without greatly changing the boundary fluxes. In the reconstruction process, one must, therefore, also be cautious and not reconstruct on material zones that are too thin optically. With the enhanced reconstruction ability of 8 experiments, the reconstruction was of much better quality with a maximum error of about 10%. This small scale test case shows the correlation between the number of experiments and reconstruction resolution.



(a) 1 Experiment (b) 8 Experiments

Figure 5. Reconstructed cross sections of centered strong absorber

Future Work:

Future work will include the analysis and effects of adding noise to true boundary solutions to simulate detector noise and bias. This will help simulate a more realistic situation where, instead of employing the true boundary solutions, synthetic detector readings could be used. Also, a multigroup analysis should also be performed based on the idea that reconstruction the energy transfer could help distinguish fission events (the only event in wish low-energy neutrons generate fast neutrons.) Finally, the models and techniques presented here will also be expanded into a transport model in the future.

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